Parallel Programming Concepts

Parallel Algorithms

Peter Tröger

Sources:

- *Ian Foster. Designing and Building Parallel Programs. Addison-Wesley. 1995.*
- *Mattson, Timothy G.; S, Beverly A.; ers,; Massingill, Berna L.: Patterns for Parallel Programming (Software Patterns Series). 1st. Addison-Wesley Professional, 2004.*
- *Breshears, Clay: The Art of Concurrency: A Thread Monkey's Guide to Writing Parallel Applications. O'Reilly Media, Inc., 2009.*

Why Parallel ?

- Amdahl‘s Law (1967)
  - P is the portion of the program that benefits from parallelization
  - Maximum speedup by N processors:
    - Maximum speedup tends to 1 / (1-P)
    - Parallelism only reasonable with small N or small (1-P)

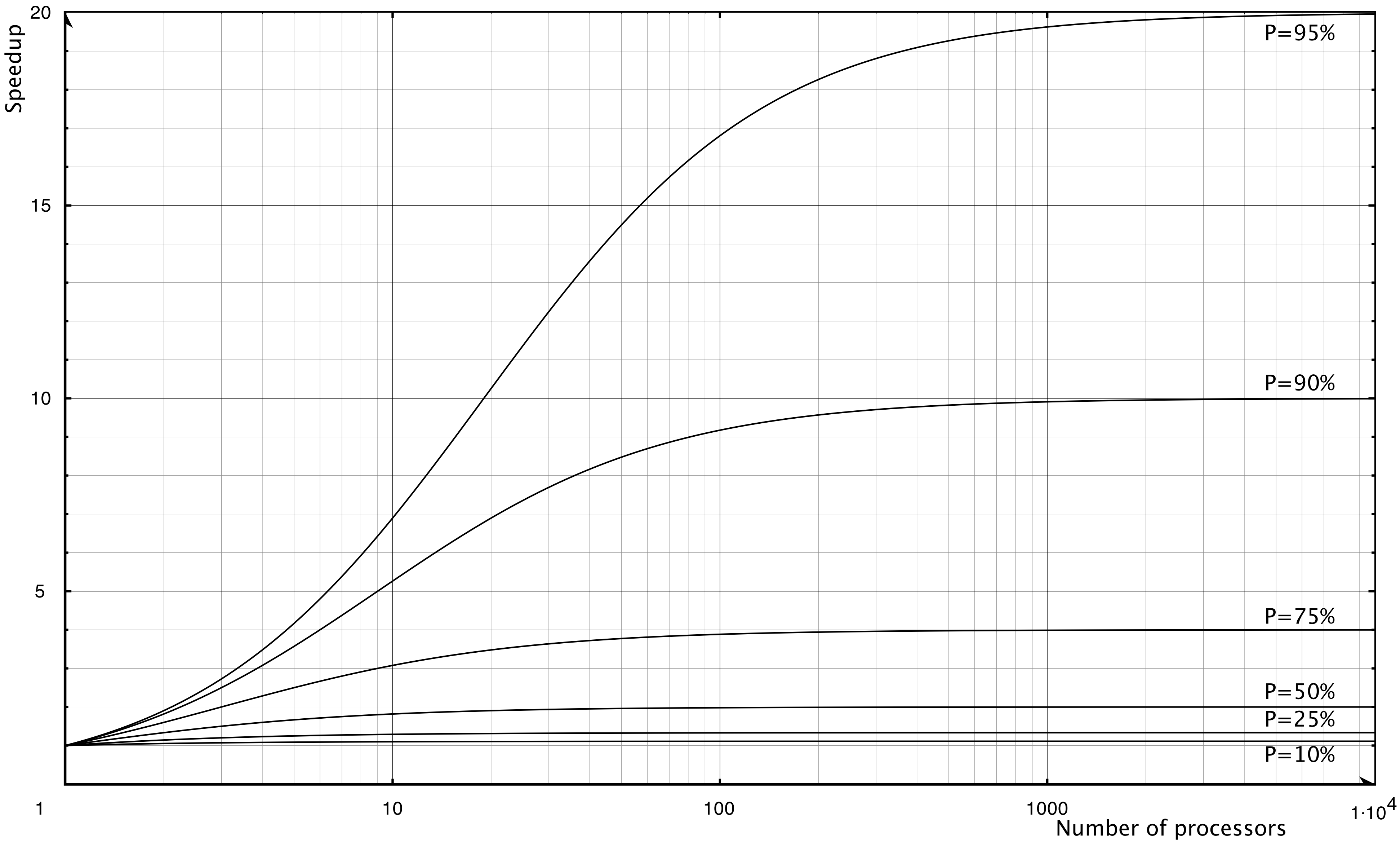
$$s = \frac{(1-P)+P}{(1-P)+\frac{P}{N}}$$

- Gustafson‘s Law

$$S(p) + N * (1 - S(p))$$

  - Let p be a measure of problem size, S(p) the time for the sequential part
  - Maximum speedup by N processors:
    - When serial function part shrinks with increasing p, speedup grows as N

Amdahl's Law



20
15
10
5
Speedup
1
10
100
1000
1·10^4
Number of processors
P=95%
P=90%
P=75%
P=50%
P=25%
P=10%

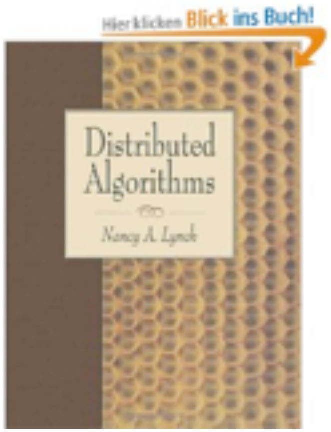
Why Parallel ?

- Karp-Flatt-Metric (Alan H. Karp and Horace P. Flatt, 1990)
  - Measure degree of code parallelization, by determining serial fraction through experimentation
  - Rearrange Amdahl's law for sequential portion
  - Allows computation of empirical sequential portion, based on measurements of execution time, without code inspection

$$S = \frac{Speed_N - \frac{1}{N}}{1 - \frac{1}{N}}$$

$$Speed_N = S + \frac{P}{N} = S + \frac{1-S}{N}$$

Distributed Algorithms [Lynch]



- Originally only for concurrent algorithms across geographically distributed processors
- Attributes
  - IPC method (shared memory, point-to-point, broadcast, RPC)
  - Timing model (synchronous, partially synchronous, asynchronous)
  - Fault model
  - Problem domain
- Have to deal with uncertainties
  - Unknown number of processors, unknown network topology, inputs at different locations, non-synchronized code execution, processor nondeterminism, uncertain message delivery times, unknown message ordering, processor and communication failures

Designing Parallel Algorithms [Breshears]

- Parallel solution must keep *sequential consistency* property
- „Mentally simulate" the execution of parallel streams on suspected parts of the sequential application
- Amount of computation per parallel task must offset the overhead that is always introduced by moving from serial to parallel code
- *Granularity*: Amount of computation done before synchronization is needed
  - **Fine-grained granularity** overhead vs.
    coarse-grained granularity concurrency
  - Iterative approach of finding the right granularity
  - Decision might be only correct only for the execution host under test
- Execution order dependency vs. data dependency

Designing Parallel Algorithms [Foster]

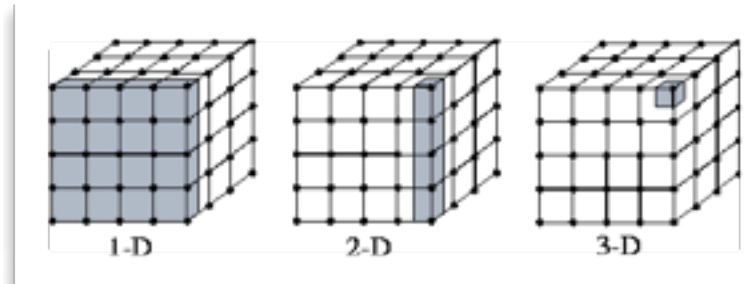
- Translate problem specification into an algorithm achieving concurrency, scalability, and locality
- Best parallel solution typically differs massively from the sequential version
- Four distinct stages of a methodological approach
  - Search for concurrency and scalability:
    - 1) **Partitioning** - decompose computation and data into small tasks
    - 2) **Communication** - define necessary coordination of task execution
  - Search for locality and other performance-related issues:
    - 3) **Agglomeration** - consider performance and implementation costs
    - 4) **Mapping** - maximize processor utilization, minimize communication
- Might require backtracking or parallel investigation of steps

Partitioning Step

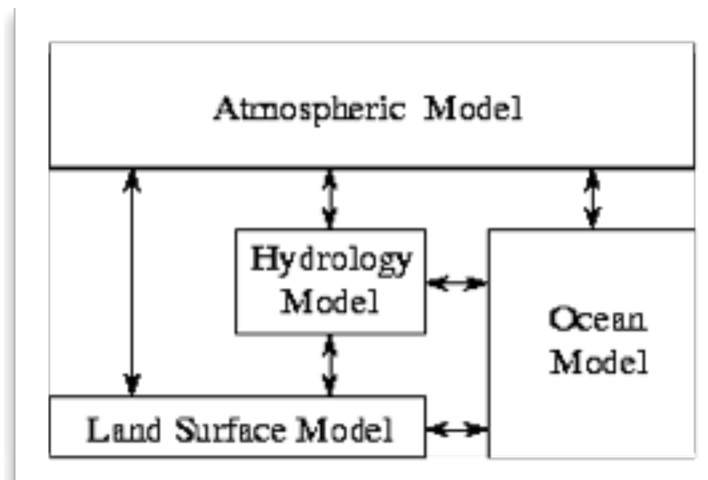
- Expose opportunities for parallel execution - fine-grained decomposition
- Good partition keeps computation and data together
  - First deal with data partitioning - *domain / data decomposition*
  - First deal with computation partitioning - *functional / task decomposition*
  - Complementary approaches, can lead to different algorithm versions
  - Reveal hidden structures of the algorithm that have potential through complementary views on the problem
- Avoid replication of either computation or data, can be revised later to reduce communication overhead
- Step results in multiple candidate solutions

Partitioning - Decomposition Types

- Domain Decomposition
    - Define small data fragments, then specify computation for them
    - Different phases of computation on the same data are handled separately
    - Rule of thumb: First focus on large or frequently used data structures



- Functional Decomposition
    - Split up computation into disjoint tasks, ignore the data accessed for the moment
    - Example: Producer / consumer
    - With significant data overlap, domain decomposition is more appropriate



Parallelization Strategies [Breshears]

- Loop parallelization
  - Reason about code behavior when loop would be executed backwards - strong indicator for independent iterations
- Produce at least as many tasks as there will be threads / cores
  - But: Might be more effective to use only fraction of the cores (granularity)
  - Computation part must pay-off with respect to parallelization overhead
- Avoid synchronization, since it adds up as overhead to serial execution time
- Patterns for data decomposition: by element, by row, by column group, by block
  - Influenced by surface-to-volume ratio

Partitioning - Checklist

- Checklist for resulting partitioning scheme
  - Order of magnitude more tasks than processors ?
    -> Keeps flexibility for next steps
  - Avoidance of redundant computation and storage requirements ?
    -> Scalability for large problem sizes
  - Tasks of comparable size ?
    -> Goal to allocate equal work to processors
  - Does number of tasks scale with the problem size ?
    -> Algorithm should be able to solve larger tasks with more processors
- Resolve bad partitioning by estimating performance behavior,
  and eventually reformulating the problem

Communication Step

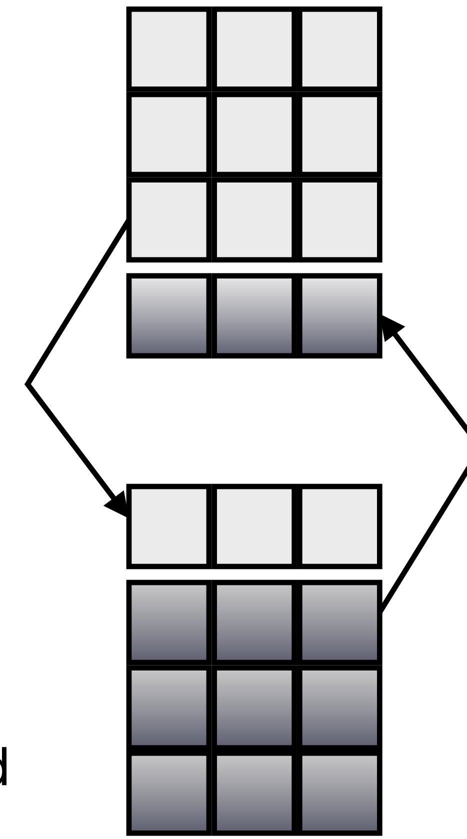
- Specify links between data consumers and data producers
- Specify kind and number of messages on these links
- Domain decomposition problems might have tricky communication infrastructures, due to data dependencies
- Communication in functional decomposition problems can easily be modeled from the data flow between the tasks
- Categorization of communication patterns
  - *Local* communication (few neighbors) vs. *global* communication
  - *Structured* communication (e.g. tree) vs. *unstructured* communication
  - *Static* vs. *dynamic* communication structure
  - *Synchronous* vs. *asynchronous* communication

Communication - Hints

- Distribute computation and communication, don‘t centralize algorithm
  - Bad example: Central manager for parallel reduction
  - *Divide-and-conquer* helps as mental model to identify concurrency
- Unstructured communication is hard to agglomerate, better avoid it
- Checklist for communication design
  - Do all tasks perform the same amount of communication ?
    -> Distribute or replicate communication hot spots
  - Does each task performs only local communication ?
  - Can communication happen concurrently ?
  - Can computation happen concurrently ?

Ghost Cells

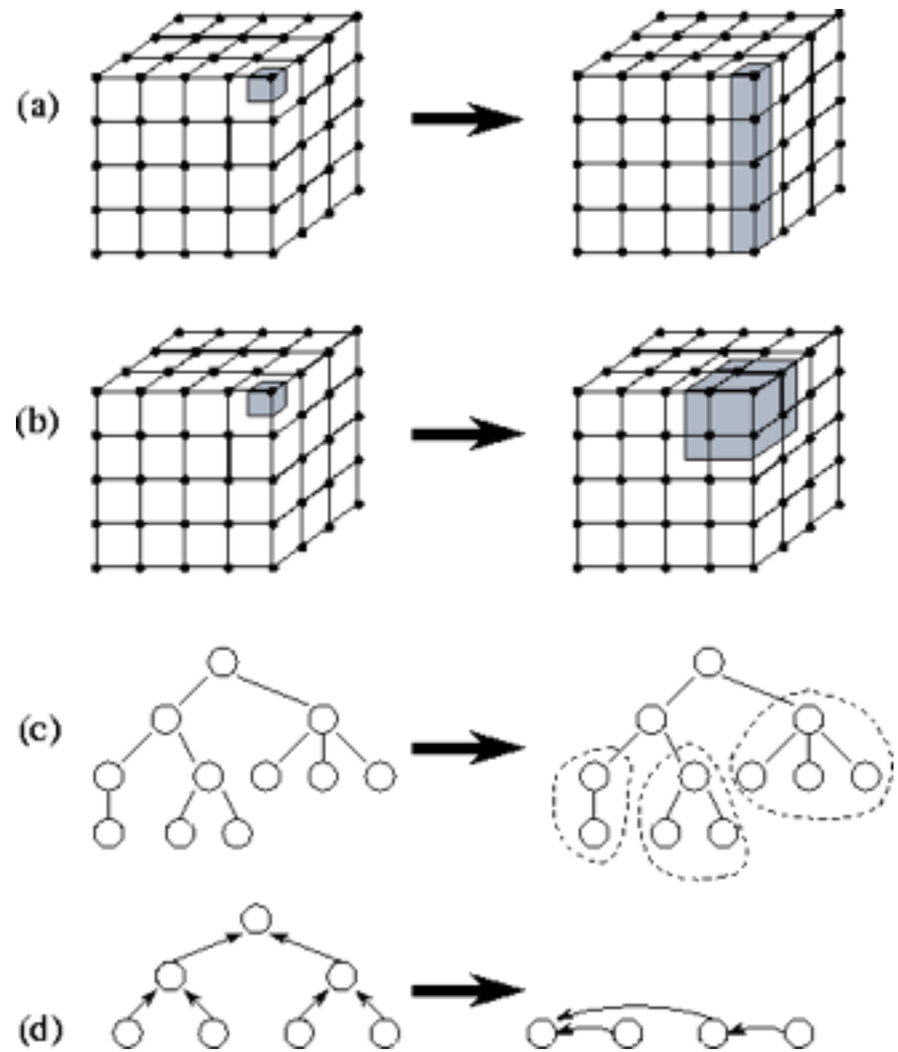
- Domain decomposition might lead to chunks that demand data from each other for their computation
  - Solution 1: Copy necessary portion of data (‚ghost cells‘)
    - Feasible if no synchronization is needed after update
    - Data amount and frequency of update influences resulting overhead and efficiency
    - Additional memory consumption
  - Solution 2: Access relevant data ‚remotely‘ as needed
    - Delays thread coordination until the data is really needed
    - Correctness („old“ data vs. „new“ data) must be considered on parallel progress



Agglomeration Step

- Algorithm so far is correct, but not specialized for some execution environment
- Revisit partitioning and communication decisions
  - Agglomerate tasks for more efficient execution on some machine
  - Replicate data and / or computation for efficiency reasons
- Resulting number of tasks can still be greater than the number of processors
- Three conflicting guiding decisions
  - Reduce communication costs by *coarser granularity* of computation and communication
  - *Preserve flexibility* with respect to later mapping decisions
  - Reduce software engineering costs (serial -> parallel version)

Agglomeration [Foster]



Agglomeration - Granularity vs. Flexibility

- Reduce communication costs by coarser granularity
  - Sending less data
  - Sending fewer messages (per-message initialization costs)
  - Agglomerate tasks, especially if they cannot run concurrently anyway
    - Reduces also task creation costs
  - Replicate computation to avoid communication (helps also with reliability)
- Preserve flexibility
  - Flexible large number of tasks still prerequisite for scalability
- Define granularity as compile-time or run-time parameter

Agglomeration - Checklist

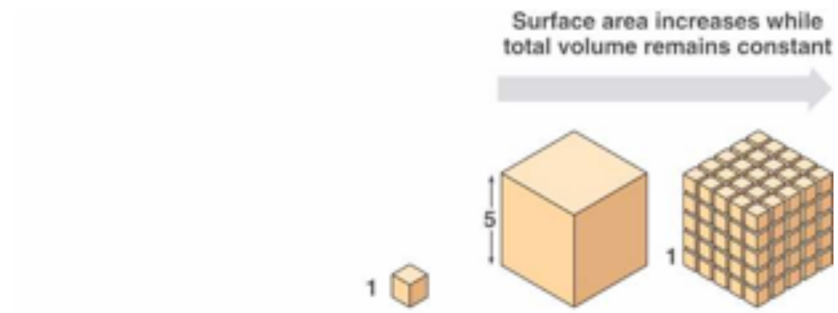
- Communication costs reduced by increasing locality ?
- Does replicated computation outweighs its costs in all cases ?
- Does data replication restrict the range of problem sizes / processor counts ?
- Does the larger tasks still have similar computation / communication costs ?
- Does the larger tasks still act with sufficient concurrency ?
- Does the number of tasks still scale with the problem size ?
- How much can the task count decrease, without disturbing load balancing, scalability, or engineering costs ?
- Is the transition to parallel code worth the engineering costs ?

Mapping Step

- Only relevant for distributed systems, since shared memory systems typically perform automatic task scheduling
- Minimize execution time by
  - Place concurrent tasks on different nodes
  - Place tasks with heavy communication on the same node
- Conflicting strategies, additionally restricted by resource limits
  - In general, NP-complete bin packing problem
- Set of sophisticated (dynamic) heuristics for *load balancing*
  - Preference for local algorithms that do not need global scheduling state

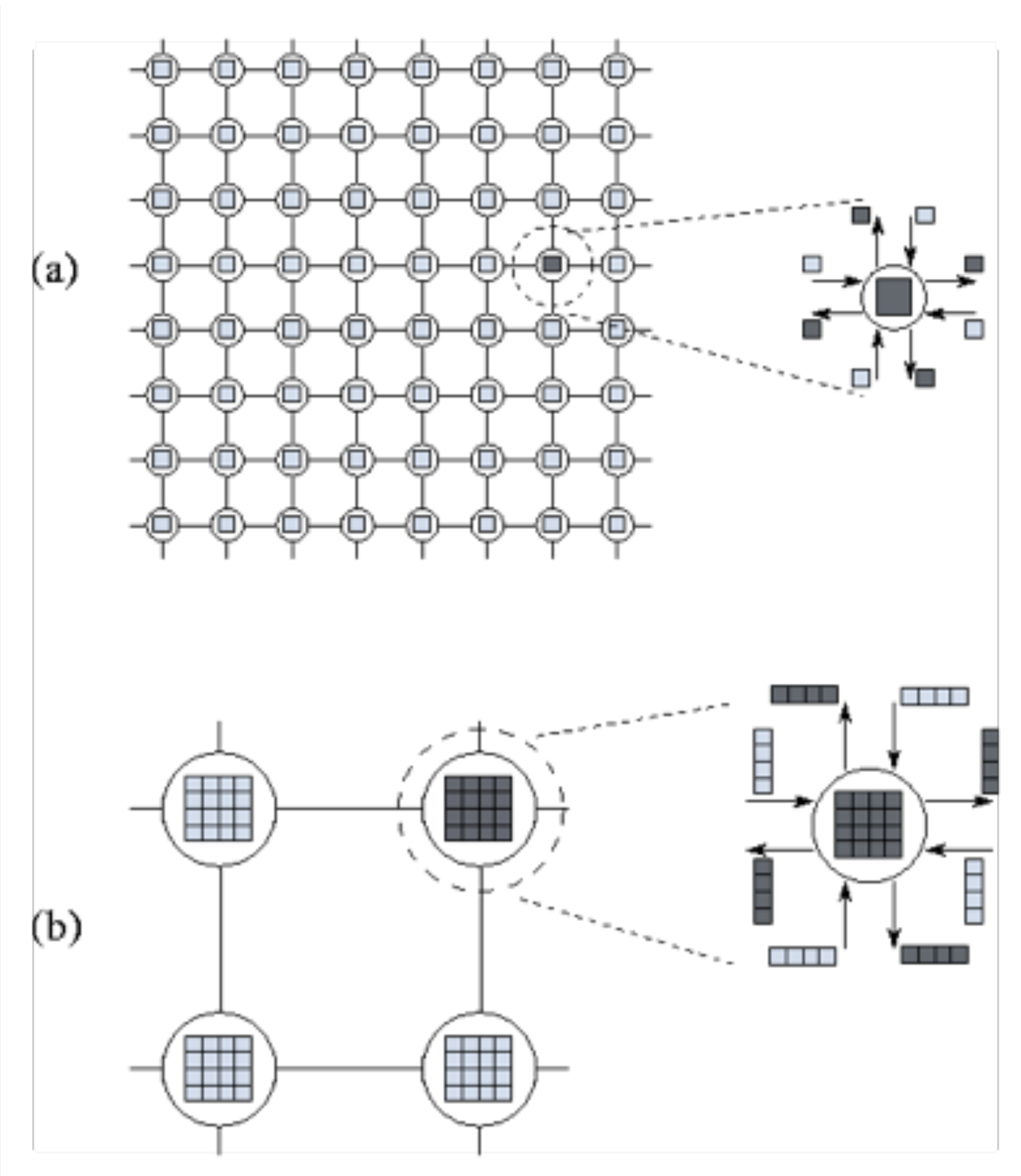
Surface-To-Volume Effect [Foster, Breshears]

- **Communication** requirements of a task are proportional to the **surface** of the data part it operates upon - amount of ‚borders‘ on the data
- **Computational** requirements of a task are proportional to the **volume** of the data part it operates upon - granularity of decomposition
- **Communication / computation ratio** decreases (good) for increasing data size per task
- Result: Better to increase granularity by agglomerating tasks in all dimensions
  - For given volume (computation), the surface area (communication) then goes down



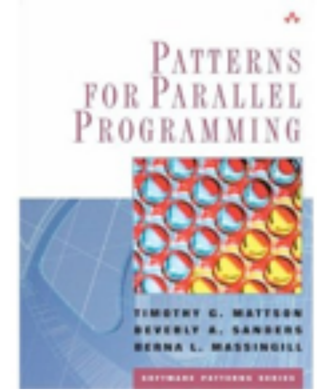
Total surface area (height × width × number of sides × number of boxes)	6	150	750
Total volume (height × width × length × number of boxes)	1	125	125
Surface-to-volume ratio (surface area / volume)	6	1.2	6

Surface-to-Volume Effect [Foster]



- Computation on 8x8 grid
- (a): 64 tasks, one point each
  - 64x4=256 communications
  - 256 data values are transferred
- (b): 4 tasks, 16 points each
  - 4x4=16 communications
  - 16x4=64 data values are transferred

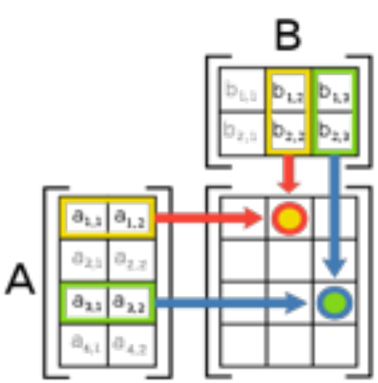
Patterns for Parallel Programming [Mattson]



- Categorization of general parallelization concepts as linear hierarchy
  - *Finding Concurrency Design Space* - task / data decomposition, task grouping and ordering due to data flow dependencies, design evaluation
    - Identify and analyze exploitable concurrency
  - *Algorithm Structure Design Space* - task parallelism, divide and conquer, geometric decomposition, recursive data, pipeline, event-based coordination
    - Mapping of concurrent design elements to units of execution
  - *Supporting Structures Design Space* - SPMD, master / worker, loop parallelism, fork / join, shared data, shared queue, distributed array
    - Program structures and data structures used for code creation
  - *Implementation Mechanisms Design Space* - threads, processes, synchronization, communication

Data Decomposition [Mattson]

- Good strategy if ...
  - ... most computation is organized around the manipulation of a large data structure
  - ... similar operations are applied to different parts of the data structure
- Data decomposition is often driven by needs from task decomposition
- Array-based computation (row, column, block), recursive structures
- In a good data decomposition, dependencies scale at lower dimension than the computational effort for each chunk
- Example: Matrix multiplication
  - C=A*B - decompose C into row blocks,
    requires full B, but only the corresponding A row block



(C) Wikipedia

Task Grouping [Mattson]

- Consider constraints for task groups, not for single items
  - Temporal dependency - Data flow from group A to group B necessary
  - Semantics - Group members have to run at the same time (fork / join)
  - Independent task groups - Clear identification for better scheduling
- Finding task groups, based on abstract constraints
  - Tasks that correspond to a high-level operation naturally group together
  - If tasks share a constraint (e.g. data), keep them as distinct group
  - Merge groups with same constraints

Data Sharing [Mattson]

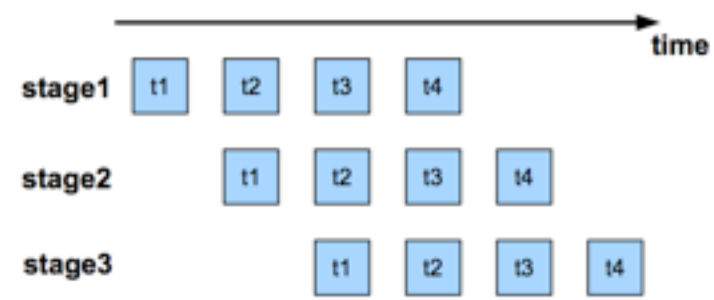
- In addition to task-local data, central dependency to shared data exists
    - Tasks might also need other tasks data, global shared read does not scale
- Analyze shared data according to its class
    - *Read-Only*: no protection overhead necessary
    - *Effectively-local*: data partitioned into independent sub sets, no locking
    - *Read-write*: global behavior must comply to a consistency model
        - *Accumulate*: Each task has local copy, final accumulation to one result
        - *Multiple-read / single-write*: Data decomposition problems
- Define abstract type with according operations
- Solve by one-time-execution, non-interfering operations, reader / writer

Algorithm Design Evaluation [Mattson]

- Minimal consideration of suitability for target platform
  - Number of processing elements and data sharing amongst them
  - System implications on physical vs. logical cores
  - Overhead for technical realization of dependency management (e.g. MPI)
- Flexibility criteria
  - Flexible number of decomposed tasks supported ?
  - Task definition independent from scheduling strategy ?
  - Can size and number of chunks be parameterized ?
  - Are boundary cases handled correctly ?

Algorithm Structure Design Space [Mattson]

- Organize by tasks
  - Linear -> Task Parallelism
  - Recursive -> Divide and Conquer (e.g. Merge Sort)
- Organize by Data Decomposition
  - Linear -> Geometric decomposition
  - Recursive -> Recursive Data
- Organize by Flow of Data
  - Regular -> Pipeline
  - Irregular -> Event-Based Coordination



Supporting Structures [Mattson]

- Program structures
  - Single-program-multiple-data (SPMB)
  - Master / worker
  - Loop parallelism
  - Fork / Join
- Data structures
  - Shared data
  - Shared queue
  - Distributed array

What‘s Not Parallel [Breshears]

- Algorithms with state that cannot be handled through parallel tasks (e.g. I/O)
- Recurrence relations - each loop run is a function of the previous one
  - Example: Fibonacci
- Reduction - take arrays of values and reduce them to a single value
  - For associative and commutative operators, parallelization is possible
- Loop-carried dependency - use results of previous iterations in loop body

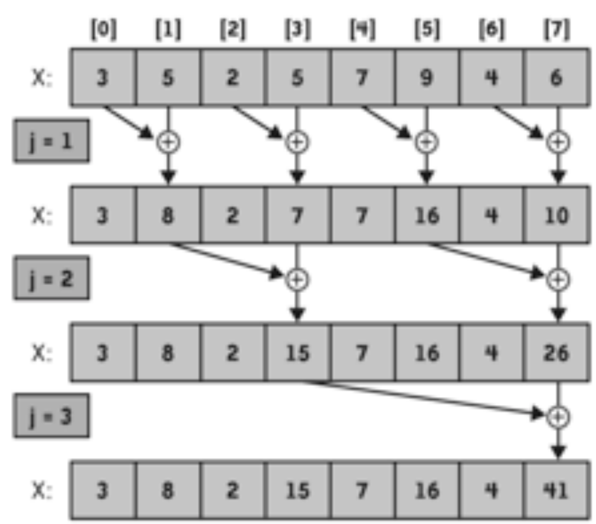
```
for (n=0; n<=N; ++n) {
    opt[n] = Sn;
    Sn *= 1.1; }
```

Parallel Algorithms and Design Patterns

- Vast body of knowledge in books and scientific publications
- Typically discussion based on abstract machine model (e.g. PRAM), to allow theoretical complexity analysis
- Rule of thumb: Somebody else is smarter than you - reuse !!
  - *Jaja, Joseph: An introduction to parallel algorithms. Redwood City, CA, USA : Addison Wesley Longman Publishing Co., Inc., 1992. , 0-201-54856-9*
  - *Herlihy, Maurice; Shavit, Nir: The Art of Multiprocessor Programming. Morgan Kaufmann, 2008. , 978-0123705914*
  - *ParaPLoP - Workshop on Parallel Programming Patterns*
  - *‚Our Pattern Language‘ (http://parlab.eecs.berkeley.edu/wiki/patterns/)*
  - *Programming language support libraries*

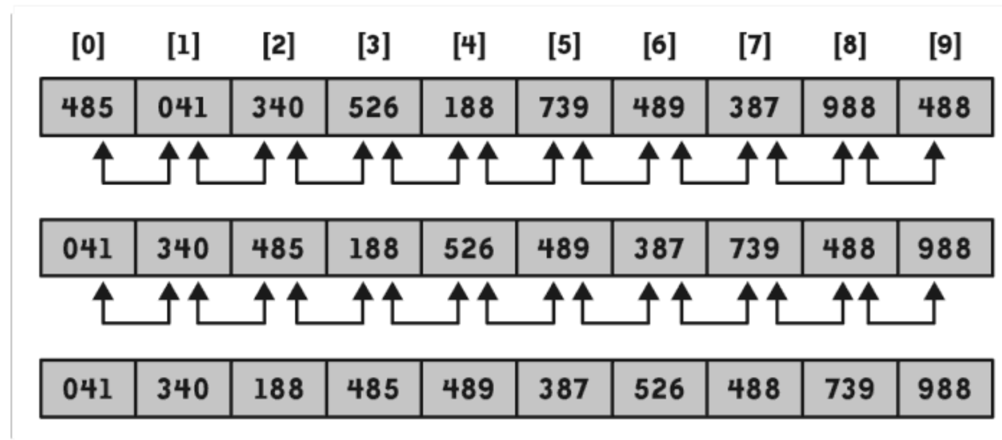
Parallel Sum [Breshears]

- Parallel sum works with every commutative and associative operation
  - Addition, multiplication, maximum, minimum,
    some logical operations, some set operations (e.g. union of sets)
  - Already supported by OpenMP `reduction` operation
- Inverted binary tree approach - Leaf nodes correspond to vector elements
  - Each addition per node is independent within the same tree level
  - In-place variant:



Parallel Sorting - Bubblesort [Breshears]

- Data decomposition
  - Execution order and data dependencies due to compare-exchange approach
- Task decomposition
  - Independent outer loop runs, while data still overlaps -> *wavefront approach*
  - Delayed start of threads, by blocking regions of the data space per thread




```
void BubbleSort(int *A, int N) {
  int i,j,temp;
  for(i=N-1; i>0; i--) {
    for(j=0; j<i;j++) {
      if (A[j] > A[j+1]) {
        temp=A[j]; A[j]=A[j+1]; A[j+1]=temp;
}}}}
```

Searching [Breshears]

```
void LinearSearch (int *A, int N,
                   int key,
                   int *position) {
  int i;
  *position=-1;
#pragma omp parallel for
  for(i=0; i<N; i++) {
   if (A[i]==key) {
      *position=i;
}}}
```

```
void BinarySearch( int *A, int lo,
                   int hi, int key,
                   int *position) {
  int mid;
  *position = -1;
  while (lo <= hi) {
    mid=(lo+hi)/2;
    if (A[mid] > key)
      hi=mid-1;
    else if (A[mid] < key)
      lo=mid+1;
    else {
      *position=mid;
      break;
}}}
```

- Search parallelization
  - Divide into non-overlapping chunks for data parallelism
- Finding the smallest key index when duplicates are allowed
  - No issue with serial version
  - Parallel version needs local result per task, and reduction step afterwards
- Global flag needed for signaling result availability to other parallel task -> granularity vs. overhead ?

Parallel Graph Algorithms [Breshears]

- Typical representation of a graph as adjacency matrix (row and columns represent node ID's, matrix value represents edge weight)
- Depth search - visit adjacent nodes, starting with the most-left unvisited leaf
  - Check row-by-row in the adjacency matrix
  - Visiting a node typically represents some computation
    (e.g. labeling,
    check for winning / losing position)
  - Serial recursive solution

```
int * visited;
int **adj;
int V;          // # nodes in graph

void visit(int l) {
  int i;
  visited[k]=1;
  // some computation with node
  for(i=0; i<V; i++) {
    if(adj[k][j])
      if(!visited[i]) visit(i);
}}

void dfsearch() {
  int k;
  for(k=0;k<V;k++) visited[k]=0;
  for(k=0;k<V;k++)
    if (!visited[k]) visit(k);
}
```

Parallel Graph Algorithms [Breshears]

- Recursive serial algorithms are tough to parallelize, so switch to iterative solution
- Concurrent solution
  - Task decomposition, model computation per unvisited node separately
  - `visited` array and `stack` become shared data structure
  - Reader / writer lock for `visited` array would be appropriate, but no performance advantage due to small critical region (no reader overlap)
  - Other extreme would be one lock per array item - state / speed tradeoff

```
int * visited;
int **adj;
int V;
stack S;

void dfsearch() {
  int i,k;
  for(k=0; k<V; k++) visited[k]=0;
  for(k=V-1; k>=0; k--) {
    push(S, k); }
  while (size(S)>0) {
    k=pop(S);
    if (!visited[k]) {
      visited[k]=1;
      // perform node operation
      for(i=V-1; i>=0; i--)
        if(adj[k][i]) push(S,i);
}}}
```

Parallel Graph Algorithms [Breshears]

```
long *visited;
long gCount=0;
stack S;

unsigned __stdcall parwindfsearch(void *pArg) {
  int i,k,willVisit=0;
  while (1) {
    WaitForSingleObject(hSem, INFINITE); // check if there are nodes on the stack
    if(gCount==V) break;                 // termination if all nodes are checked
    k=pop(S);
    if (!InterlockedCompareExchange(&visited[k], 1L, 0L)) {   // grab node safely
      willVisit=1;
      InterlockedIncrement(&gCount); }
    if (willVisit) {                     // check a complete row in this thread
      // perform node computation
      for(i=V-1;i<=0;i--) {
        int semCount=0;                  // use variable semCount to update
        if (adj[k][i]) {                 // number of stack nodes only ones
          push(S, i);
          semCount++; }
        if (semCount) ReleaseSemaphore(hSem, semCount, NULL); }
      willVisit=0;
      if (gCount==V) SetEvent(tSignal);  // trigger external ReleaseSemaphore,
  }}                                     // in case all threads wait on an
  return 0;}                             // empty stack
```