State-Based Dependability Modeling

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M. A. Marsan, “Stochastic petri nets: An elementary introduction,” in Advances in Petri Nets, pp. 1–29, Springer, 1989.

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Malhotra, Manish, and Kishor S. Trivedi. "Power-hierarchy of dependability-model types." Reliability, IEEE Transactions on 43.3 (1994): 493-502.

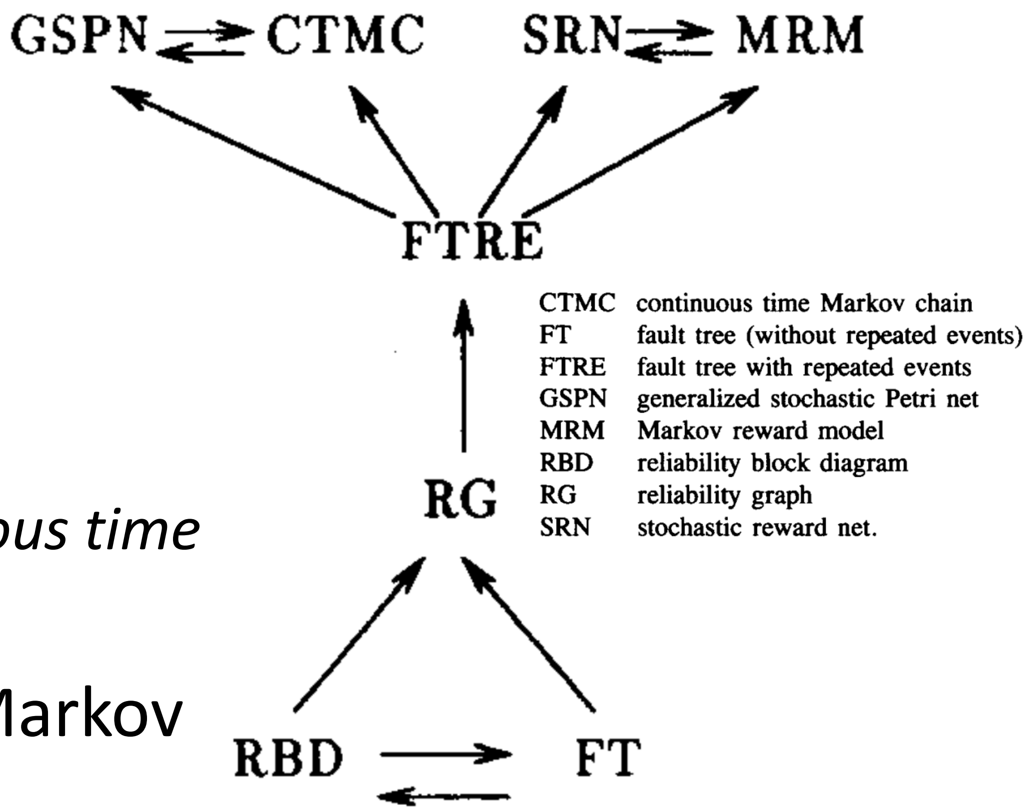
Dependability Modeling

RECAP

- Use a formalism to model system dependability
  - Quantify dependability attributes of components
    - Calculate system availability/reliability
    - Based on a set of data and assumptions - the *availability model*
  - Most models expose the same expressiveness
  - Each formalism allows to focus on certain aspects
  - **Component-based** models: Reliability block diagrams, fault trees
  - **State-based** models: Markov chains, petri nets
- System understanding evolved from hardware to software to IT infrastructures
  - Example: Organization management influence on business service reliability
    - Information Technology Infrastructure Library (ITIL)
    - CoBiT(Control Objectives for Information and related Technology)

Structural vs State-Based Dependability Models

- **Structural** / combinatorial models:
  - Focus on static system structure
  - High-level graphical modelling
  - Mapping components to model elements
- **State-based** / Markov models:
  - Focus on dynamic behaviour
  - Notion of *stochastic distributions in continuous time*
  - Can be solved analytically or simulated
- Structural models are often mapped to Markov models for quantitative analysis



Malhotra, Manish, and Kishor S. Trivedi. "Power-hierarchy of dependability-model types."

State-based models

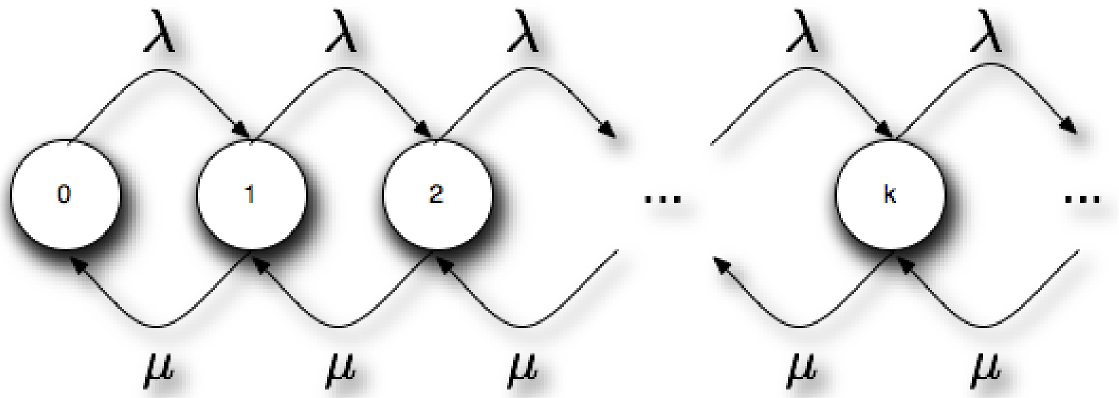
- Component-based models work well if failure events are stochastically independent
  - But: Catastrophic events destroy multiple components
- State-based models focus on failure states of the system
  - Can handle transitions between failure states
  - Independent of the system structure
- **Analytical** solution
  - Demands independent failures, constant failure rates, (exponential distribution)
- Solution through **simulation**
  - State model is simulated to estimate the resulting dependability metrics
  - Arbitrary failure event distributions, approximations, long simulation time

State Transition Diagrams

- Modelling approach typically used for queueing systems
- Assumptions
  - **Homogeneous workload assumption**
    All request are indistinguishable, so only their sum counts
  - **Operational equilibrium**
    Number of requests in the system is the same at the start and end of investigation
    - May vary in the interval, but average throughput is constant
    - Number of departures tends to approach the number of arrivals - „all forces on the object are balanced“
  - **Memoryless assumption**
    Server state is a single parameter - number of processed requests

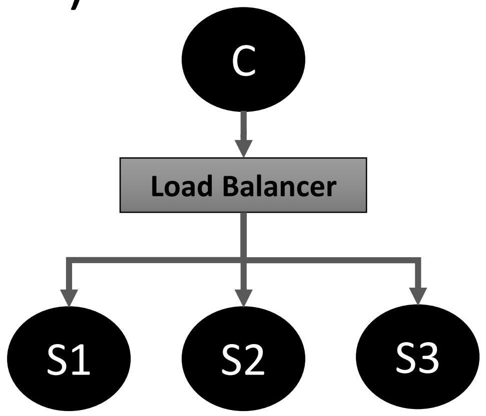
State Transition Diagrams

- Transitions between states happen at some *rate*
  - Arrival rate λ (transitions / sec), request completion rate μ (transitions / sec)
- **Flow-In Flow-Out** principle
  - Operational equilibrium ensures that transitions into the state are equal to transitions out of that state
  - Not relevant how this state was reached and how long it stays in it



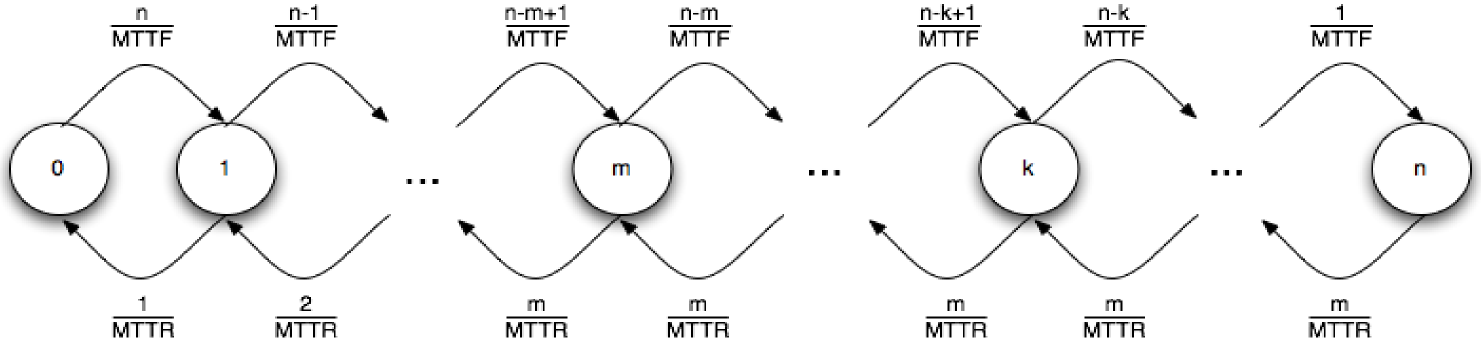
State Transition Diagrams: Application

- System of n parallel servers which ‚arrive‘ at repair situation (i.e., fail)
  - Maximum number *m* of parallel repair activities
  - Maximum *k-out-of-n* servers are allowed to be failed
  - Arrival rate == failure rate
  - Completion rate == repair rate
  - State: number of servers down
  - Transitions: a component failure or a component back in operation



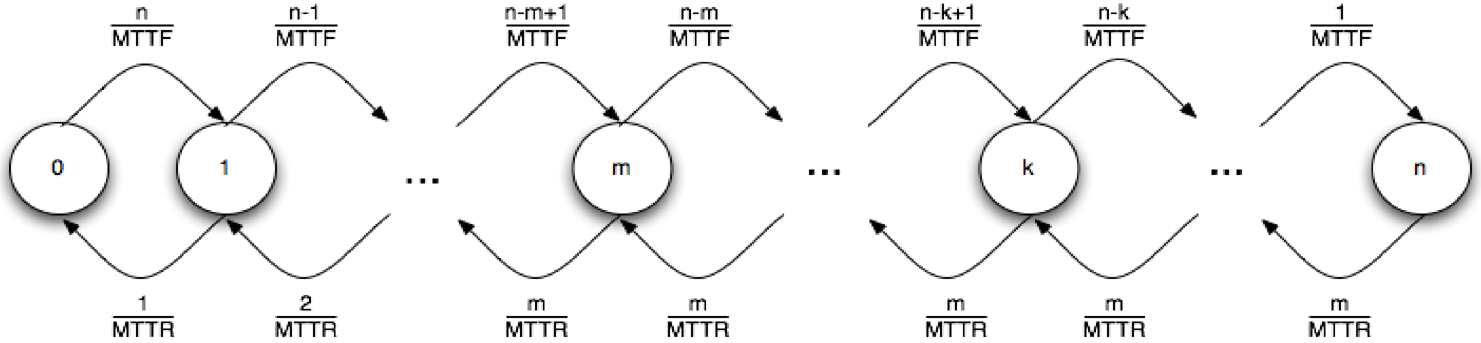
$$\lambda_k = \frac{n-k}{MTTF}$$

$$\mu_k = \begin{Bmatrix} k/MTTR & k = 1, ..., m \\ m/MTTR & k = m+1, ..., n \end{Bmatrix}$$



State Transition Diagrams: Analysis

Probability that k servers are down	p_k
Site availability	$A = 1 - p_n$
Average number of working servers	$N = \sum_{k=0}^{n-1} (n-k) \times p_k$
Probability of more than k servers down	$P = \sum_{j=k+1}^{n} p_j$



Markov Chains

- Discrete random process, usually drawn as state transition diagram
- **Markov property:** next step depends only on the current step

$$P(X_{n+1}|X_1, X_2, \dots, X_n) = P(X_{n+1}|X_n)$$

  - Impossible to predict future states, but useful for statistical properties
  - Finite state space (chain), transitions with probabilities, initial state probabilities
- **Transient state**: probability > 0 to not return to this state (finite number of visits)
- **Recurrent state**: probability of 1 to return to this state after unspecified time t
  - *Mean recurrence time* can be used as MTTF metric
- **Time-homogeneous Markov chains:** transition probabilities/rates do not change in time

$$P(X_{n+1} = x|X_n = y) = P(X_n = x|X_{n-1} = y)$$

Markov Chains: Time Model

Discrete-time Markov chain (DTMC)

- State changes after fixed time intervals
- Discrete parameter space, discrete state space
- System is in exactly one state
- Transition to next state depends on *transition probability* at t
- Probability transition matrix
  - Rows: flow out of that state
  - Columns: flow into the state
  - Rows and columns sum up to 1

Continuous-time Markov chain (CTMC)

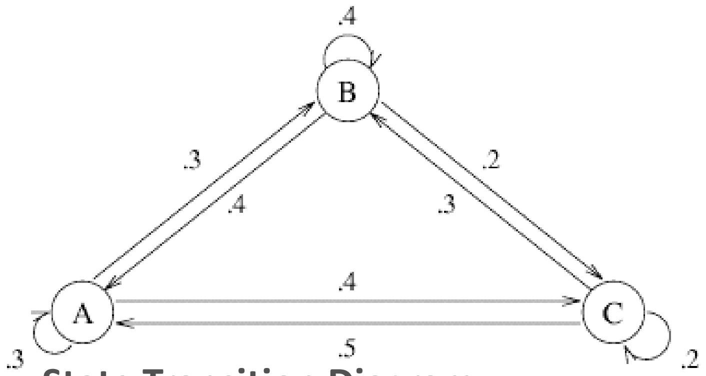
- State changes at any point in time
- Continuous parameter space, discrete state space
- Transition to next state after spending some time in a state (holding time)
- *Transition rates* instead of probabilities
- Transition rate / generator matrix Q
  - q_{ij} : rate departing from i and arriving in j
  - q_{ii} : -(total rate out of i) → no state change
  - Rows sum up to 0

Markov Chains: DTMC

- Each row sum of the transition matrix is 1
- For each step: apply transition matrix to state probability vector

$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} .3 & .3 & .4 \\ .4 & .4 & .2 \\ .5 & .3 & .2 \end{bmatrix} \end{array} = S \qquad q_{i,j} = \lim_{\Delta t \to 0} \frac{P\{X_{t+\Delta t} = j \,|\, X_t = i\}}{\Delta t} \qquad i \neq j$$

Transition Matrix



State Transition Diagram

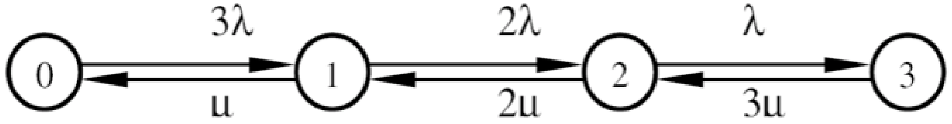
Initial state probabilities (distribution vector) $\begin{bmatrix} .\overline{3} & .\overline{3} & .\overline{3} \end{bmatrix} \begin{bmatrix} .3 & .3 & .4 \\ .4 & .4 & .2 \\ .5 & .3 & .2 \end{bmatrix} = \begin{bmatrix} .4 & .\overline{3} & .2\overline{6} \end{bmatrix}$

$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} .41 & .33 & .26 \\ .38 & .34 & .28 \\ .37 & .33 & .30 \end{bmatrix} \end{array} = S^2$$

Probability Matrix after 2 steps

(C) Tamara Lynn Anthony

Dependability Modelling with CTMCs



- **State**: represents a particular error state
  - E.g., number of failed components at any given time
- **Transition**: assigned with component failure rate
  - *Time-homogeneous* process: failure / repair rates do not change over time
  - Failure / repair events are stochastically independent, process is memory-less
- Each row sum is 0
  - Probability mass flowing out of a state will go to some other state
- **Stationary Distribution**: the probability distribution to which the chain converges after a long time
  - E.g., the availability distribution

Example

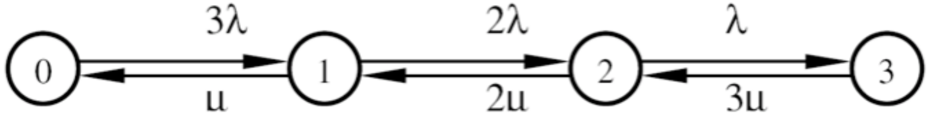
Consider a *k-out-of-n* system with *n* components.

- Their failure rates are distributed following the bathtub curve
- Their repair rates are exponentially distributed

- Would (and can) you model this system using
  - Time-homogenous DTMC?
  - Time-homogenous CTMC?
  - Time-inhomogenous CTMC?
- In a Markov chain modelling this system, which are recurring states?

Example: Availability Analysis

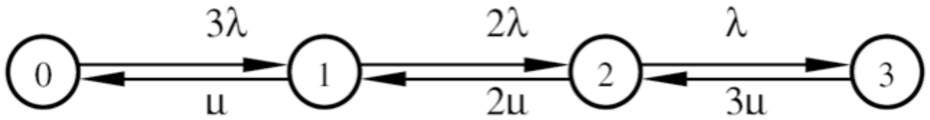
- Interested in **steady-state availability** of the system
  - Interpretation as steady-state probability for the system being operational at t
  - Derived from **probability vector**: steady-state probabilities for the system being in one of the failure states after a number of steps
- “Static” steady-state availability computable if probabilities are in **equilibrium**
  - Probability for leaving state is similar to probability for going into that state - probability mass is evenly distributed
  - Typically achieved after a high number of steps



$$Q = \begin{pmatrix} -3\lambda & 3\lambda & 0 & 0 \\ \mu & -\mu - 2\lambda & 2\lambda & 0 \\ 0 & 2\mu & -2\mu - \lambda & \lambda \\ 0 & 0 & 3\mu & -3\mu \end{pmatrix}$$

Example: 2-out-of-3 System

$$Q = \begin{pmatrix} -3\lambda & 3\lambda & 0 & 0 \\ \mu & -\mu - 2\lambda & 2\lambda & 0 \\ 0 & 2\mu & -2\mu - \lambda & \lambda \\ 0 & 0 & 3\mu & -3\mu \end{pmatrix}$$



1. Balance equations (steady-state equilibrium criterion):

Equilibrium: P(leaving s_0) = P(entering s_0) $3\lambda s_0 = \mu s_1$

$$3\lambda s_0 + 2\mu s_2 = \mu s_1 + 2\lambda s_1$$
$$2\lambda s_1 + 3\mu s_3 = 2\mu s_2 + \lambda s_2$$
$$\lambda s_2 = 3\mu s_3$$
$$s_0 + s_1 + s_2 + s_3 = 1$$

solve for s_i

$$s_0 = \frac{\mu}{3\lambda} s_1$$
$$s_2 = \frac{\lambda}{\mu} s_1$$
$$s_3 = \frac{\lambda^2}{3\mu^2} s_1$$

$$s_1 = \frac{3\mu^2\lambda}{(\mu+\lambda)^3}$$

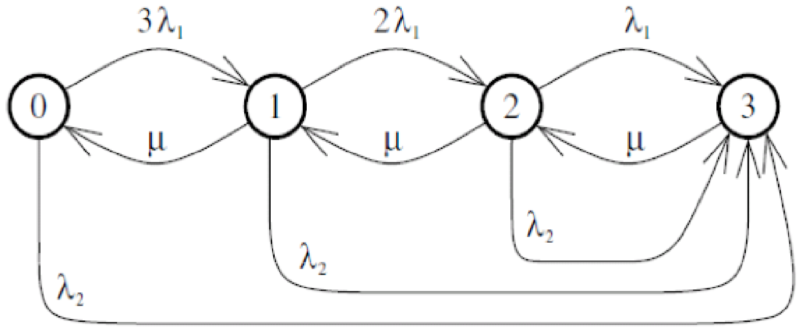
2. Compute per-state steady-state probabilities:

$$s_0 = \frac{\mu^3}{(\mu+\lambda)^3}; s_1 = \frac{3\mu^2\lambda}{(\mu+\lambda)^3}; s_2 = \frac{3\mu\lambda^2}{(\mu+\lambda)^3}; s_3 = \frac{\lambda^3}{(\mu+\lambda)^3}$$

3. 2-out-of-3 availability:

< 2 failed nodes: $A = s_0 + s_1 = \frac{\mu^2(\mu + 3\lambda)}{(\mu+\lambda)^3} = 3a^2 + 2a^3 \quad a = \frac{\mu}{(\mu+\lambda)}$

Markov Chains: Complexity



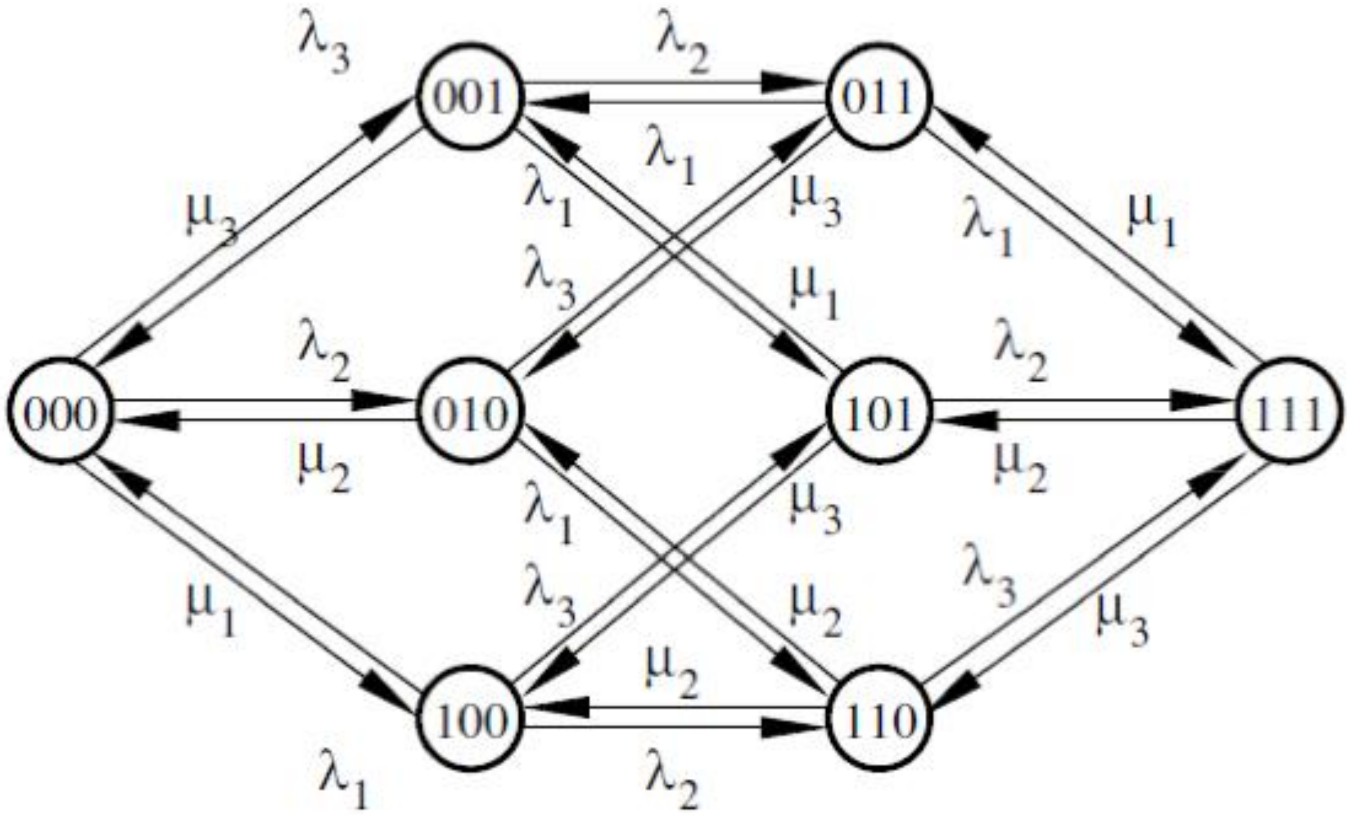
- Resulting formula equals result from Boolean investigation
- Markov chains also support non-independent events
  - Common cause failures
- Markov chains sizes grow *exponentially* with their number of components - which is bad

  → Divide-and-conquer: decompose and aggregate chain parts
  - **Structural decomposition:** consider a system as set of independent subsystems
  - **Behavioral decomposition:** assume time constants for some fault occurrences and handling processes based on criticality - e.g. fault in parked airplane

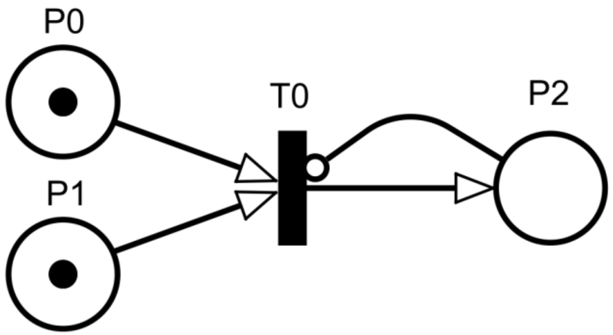
Markov Chains: Complexity

3-component model, where each component has its own failure and repair rate

→ 2^3 states



Petri Nets



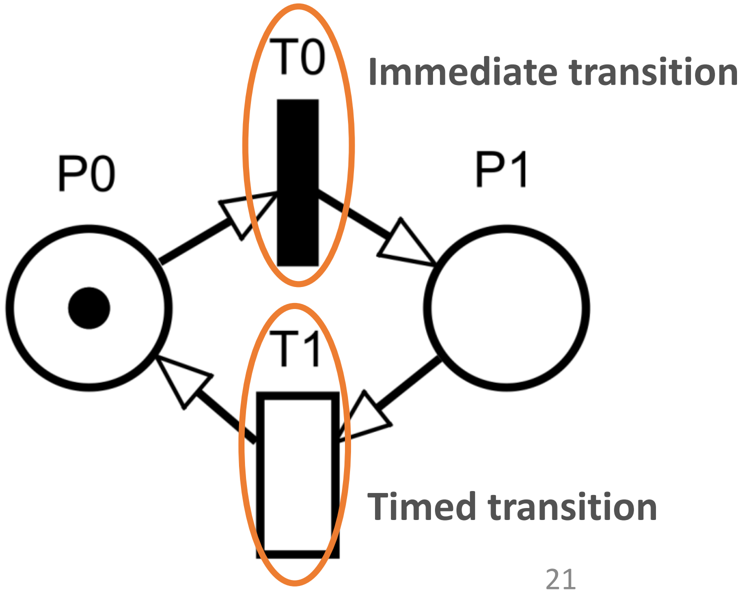
- Modelling language for concurrent, distributed systems
- Bipartite graph
  - **Places** contain **tokens** (marking) → *state*
  - **Transitions** consume & produce tokens → *behaviour*
- Simultaneous enabling of multiple transitions: *concurrent behaviour*
- Transition firing: consume tokens from input places, produce tokens in output places
  - Places: *pre/post-conditions* for state changes
  - If all input places contain tokens, a transition is *enabled*
    - Necessary number determined by arc cardinality
  - **Inhibitor arcs** disable transitions if tokens lie in their origin places
- **Conflict**: When two transitions need the same token, only one can fire
  - Resolved by priorities (absolute) or weights (randomized)
  - E.g., competing for resources

Petri Nets – Conceptual Mapping

Input Places	**Transitions**	**Output Places**
Required Resources	Task	Freed Resources
Input Data	Computations	Output Data
Input Signals	Signal Processing	Output Signals
Buffers / Registers	Processor	Buffers / Registers

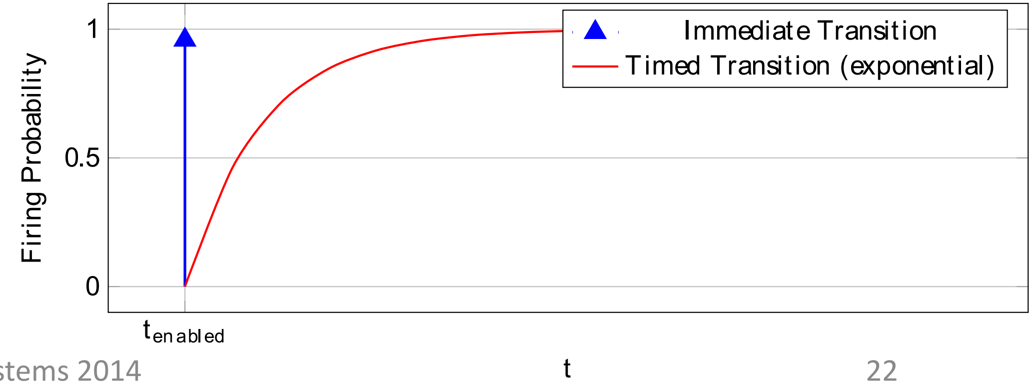
Stochastic Petri Nets

- Extend petri nets by stochastic temporal properties
  - Delayed transition firing
  - Temporal properties allow to study quantitative, time-dependent metrics
    - E.g., MTTF
  - Event propagation can be modelled in time, not just logically
    → Increased expressiveness
- **Generalized Stochastic Petri Nets** (GSPN)
  - Immediate transitions: fire immediately
  - Timed transitions: fire with stochastic delay
  - Model in *continuous time*
    - Marking corresponds to a continuous probability distribution
    - For simulation, *time discretization* becomes necessary

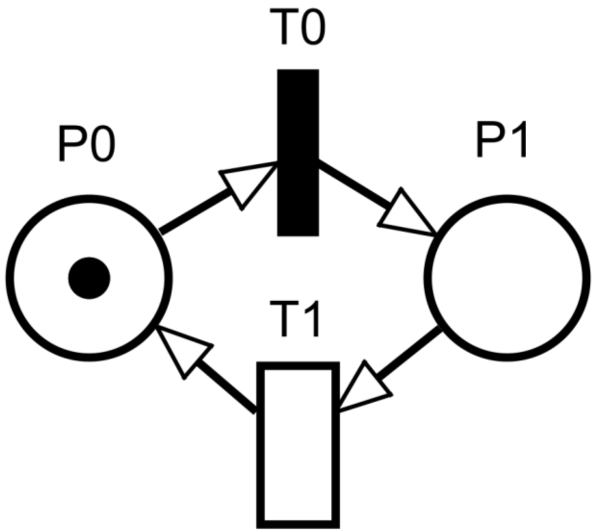


Stochastic Petri Nets: Transitions

- **Immediate transitions**
  - Model logical inter-dependencies, e.g. error propagation chains
  - Fire ‘in no time’, *0-Dirac distribution*
- **Timed transitions**
  - Model stochastic behaviour, e.g. random component failure
  - Delayed firing, defined by probability distribution in continuous time
  - For GSPN: *exponential distribution*
  - Firing policies: when to sample the delay?
    - Race with enabling memory: at enabling time
    - Race with age memory: at firing time

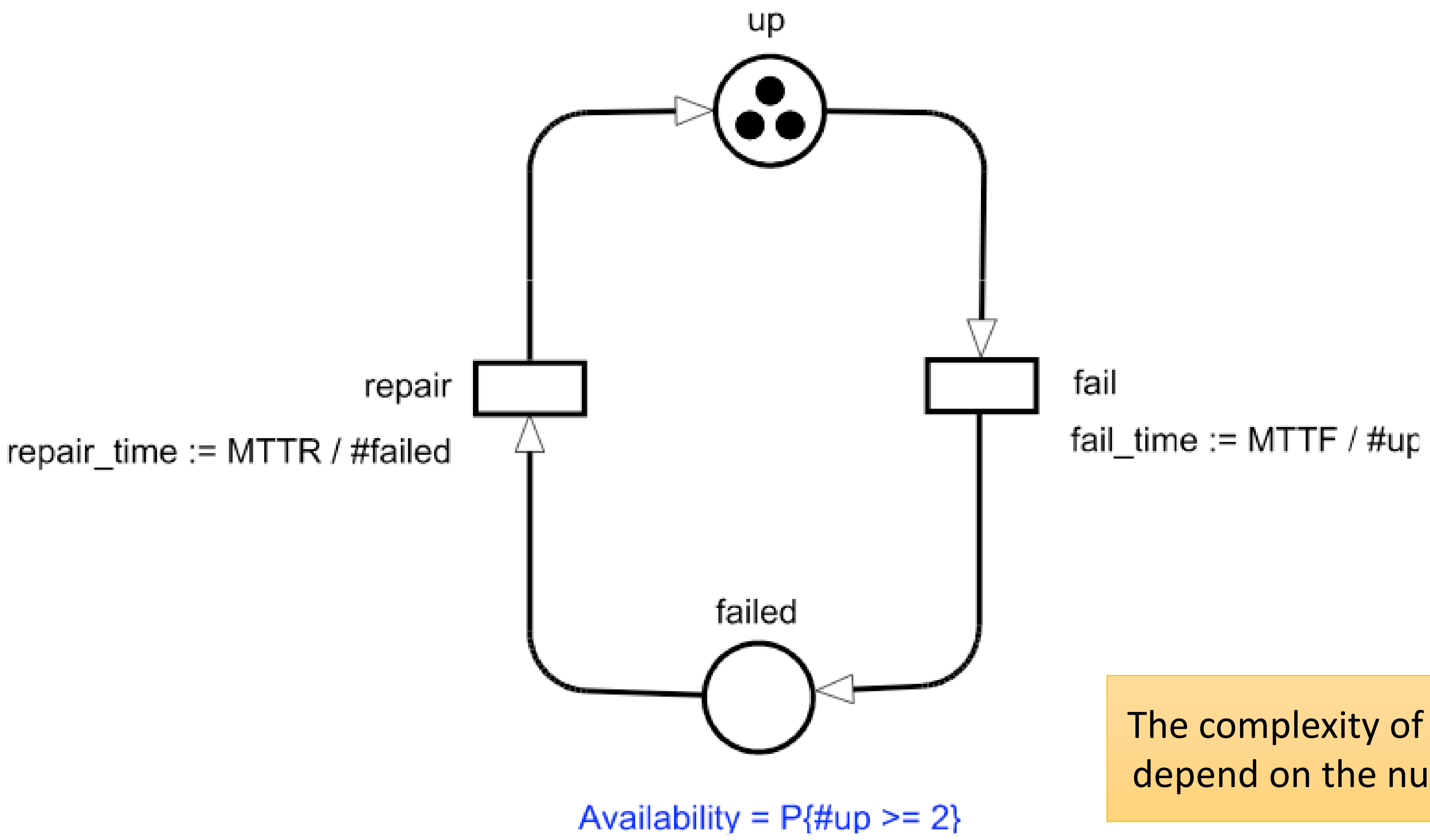


Stochastic Petri Nets – Properties



- **Reachability set**
  - Contains all possible markings reachable from the initial marking
  - Analysis questions:
    - Can some system state (e.g. an error state) be reached at all?
    - Does a firing sequence exist, that transforms M0 to M?
- **Vanishing marking (GSPN)**
  - A marking that is abandoned again at once, due to immediate transition firing
  - Probability of observing this marking: 0 (continuous time)
- **Tangible marking (GSPN)**
  - A marking that the net remains in for some time
- **Reduced Reachability graph**
  - Graph of *reachable tangible markings* from the initial marking
- **Boundedness**
  - A place is *k-bounded* if for every reachable marking, the number of tokens in it does not exceed k
  - A net is k-bounded if all places are k-bounded
  - Useful for modeling limited (bounded) resources

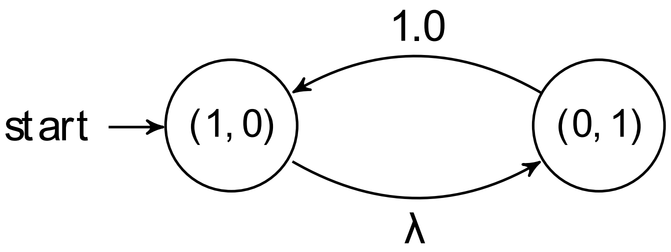
Example: 2-of-3 System

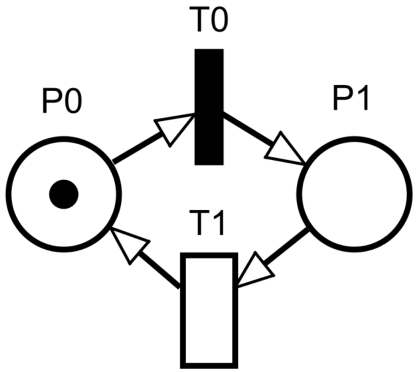


The complexity of the petri net does not depend on the number of components!

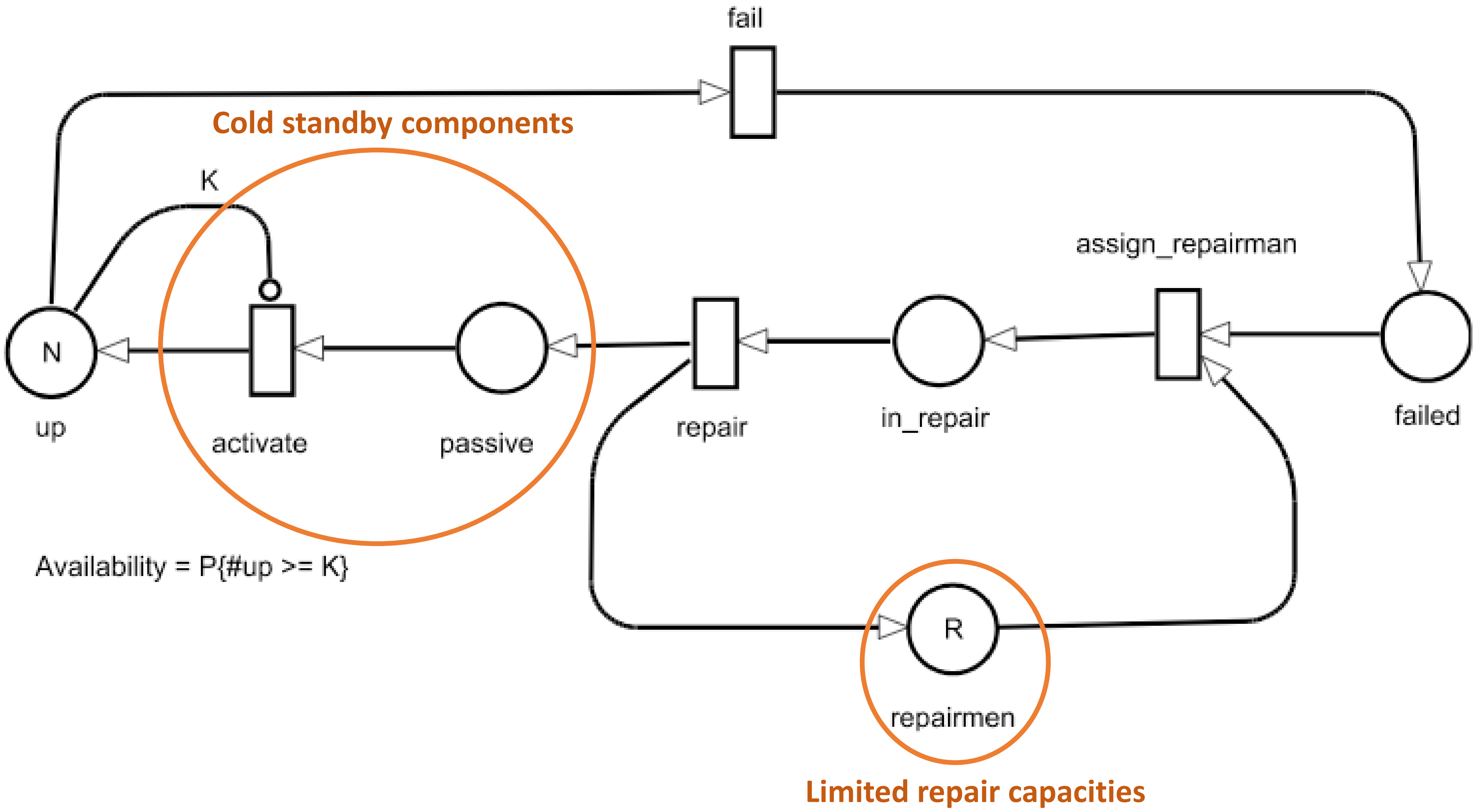
Stochastic Petri Nets vs Markov Chains

- **Reachability graphs** of GSPN are isomorphic to CTMC
- GSPN → "compact representation" of a CTMC
  - CTMC: one node per state (exponential growth with #components)
  - GSPN: one marking per state (linear growth with #components)
- GSPN simulation
  - Traverse underlying CTMC at random
  - No need to generate all states beforehand

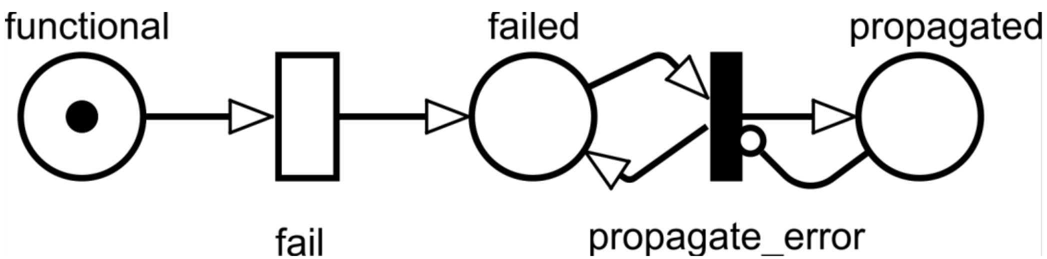




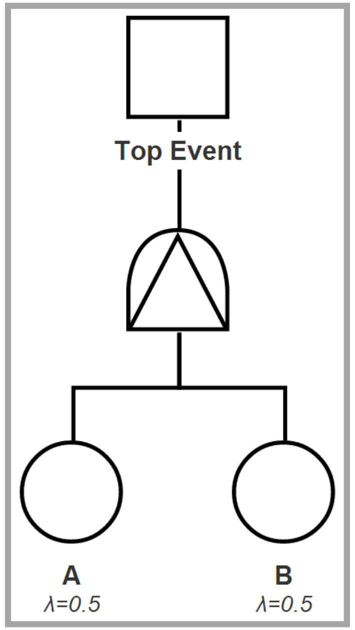
Example: K-of-N with Standby and Repairmen

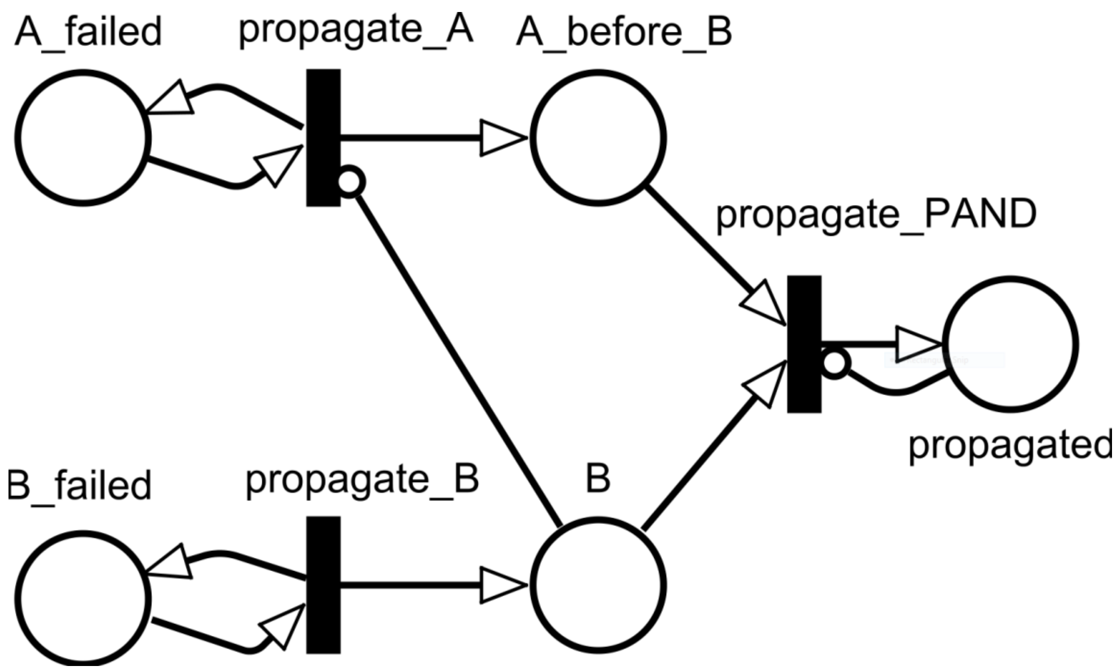


Example: Priority AND → Stochastic Petri Net



Basic Event

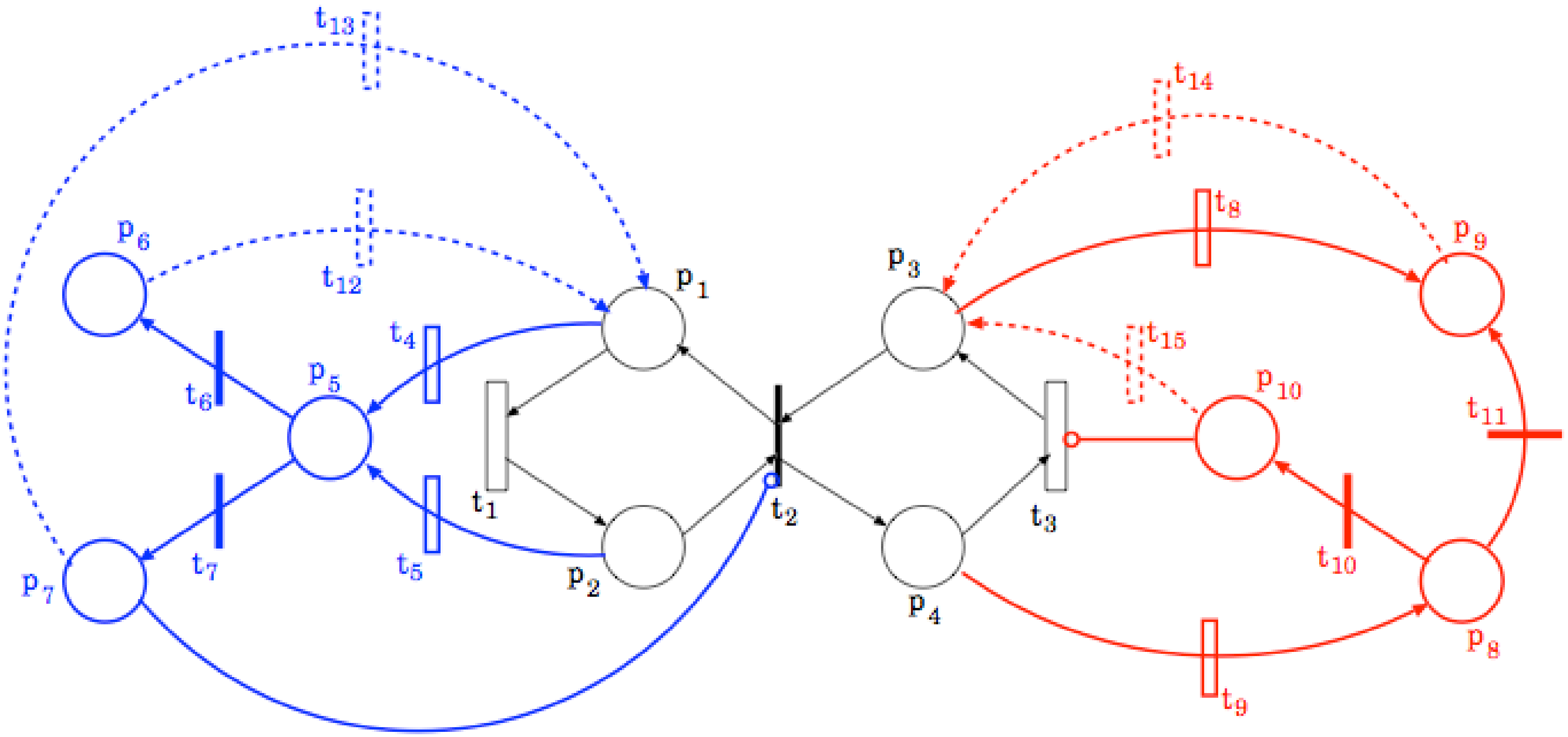




PAND Gate

Example: Parallel System with Input Buffer

p_1	Free buffer stage	
p_2	Occupied buffer stage	
p_3	Idle unit	
p_4	Active unit	
p_5	Failed buffer stage	
p_6	Recovered buffer stage failure	
p_7	Unrecovered buffer stage failure	
p_8	Failed active unit	
p_9	Recovered unit failure	
p_{10}	Unrecovered unit failure	
		firing rate
t_1	Buffer stage becomes occupied	λ
t_2	Transfer from buffer to unit	*immed.*
t_3	Unit ends a task	$m_4\,\mu$
t_4	Free buffer stage fails	$m_1\,\gamma_4$
t_5	Occupied buffer stage fails	$m_2\,\gamma_5$
t_6	Buffer stage failure is recovered	v_B
t_7	Buffer stage failure is not recovered	$(1 - v_B)$
t_8	Idle unit fails	$m_3\,\gamma_8$
t_9	Active unit fails	$m_4\,\gamma_9$
t_{10}	Unit failure is not recovered	$(1 - v_U)$
t_{11}	Unit failure is recovered	v_U
t_{12}	Repair of recovered buffer stage	ρ_{12}
t_{13}	Repair of unrecovered buffer stage	ρ_{13}
t_{14}	Repair of recovered unit	ρ_{14}
t_{15}	Repair of unrecovered unit	ρ_{15}



Light lines: Fault free operation
Heavy lines: Failures
Dotted lines: Repairs

Petri Net Simulation vs Analysis

Computational analysis

- *Compute* static properties of the net
- Probability of an event defined through place markings
  - Add up the probabilities of all markings in which the condition corresponding to the event definition holds true
- Requires construction of reachability graph → **state space explosion**
- Additional challenges
  - Transition guard functions
  - Non-exponential distributions

Simulation

- Execute the model to randomly *explore* the state space
- Play the "token game" many times (Monte Carlo approaches)
- Challenges
  - **Rare event simulation**: small failure rates → importance sampling
  - Random number generation
  - Verification of results (statistical tests)

Petri Net Simulation: Token Game

```
bool PetriNetSimulation::simulationStep(PetriNet* pn, int tick)
{
    bool immediateCanFire = true;
    while (immediateCanFire)
        tryImmediateTransitions(pn, tick, immediateCanFire);

    tryTimedTransitions(pn, tick);

    vector<TimedTransition*> toRemove;
    for (TimedTransition* tt : pn->m_inactiveTimedTransitions)
    {
        if (tt->tryUpdateStartupTime(tick))
        { // tt has become enabled -> start its timer
            toRemove.emplace_back(tt);
            pn->updateFiringTime(tt); // register in event queue
        }
    }
    for (TimedTransition* tt : toRemove)
        pn->m_inactiveTimedTransitions.erase(tt);

    return !pn->markingInvalid();
}
```

immediate transitions
(error propagation)

timed transitions
(basic events)

update event queue

Rare Event Simulation: Importance Sampling

- Problem: naïve simulation is inefficient for very rare events
  - Such as simulating components with low failure rates
  - Monte Carlo methods with moderately many rounds have *high variance*
  - Many rounds needed to achieve a desired confidence level

Importance Sampling: Compute $\mathrm{p} = E\big(\phi(X)\big)$, where $\phi(X)$ is a desired dependability metric, and X a rare random variable

- But sample from a different, not rare, distribution!
- Instead of sampling from $PDF(x)$, sample from $PDF^*(x)$
  - $PDF(x) > 0 \Rightarrow PDF^*(x) > 0$
  - Likelihood ratio: $w(x) = \frac{PDF(x)}{PDF^*(x)}$

→ $\mathrm{p} = E(\phi(X^*) * w(x))$

Importance Sampling

$$\mathcal{E}(\phi(x)) = \frac{1}{n}\sum_{i=1}^{n} \phi(x_i) \quad , x_i \sim p$$

$$\approx \int \phi(x)\, p(x)\, dx = \int \phi(x) \underbrace{\frac{p(x)}{q(x)}}_{\omega(x)} q(x)\, dx$$

$$\approx \frac{1}{n}\sum_{i=1}^{n} \phi(x_i) \cdot \omega(x) \quad , x_i \sim q$$

$$\Rightarrow \underset{x \sim p}{E}(\phi(x)) = \underset{x \sim q}{E}(\phi(x') \cdot \omega(x))$$

Reliability Tools

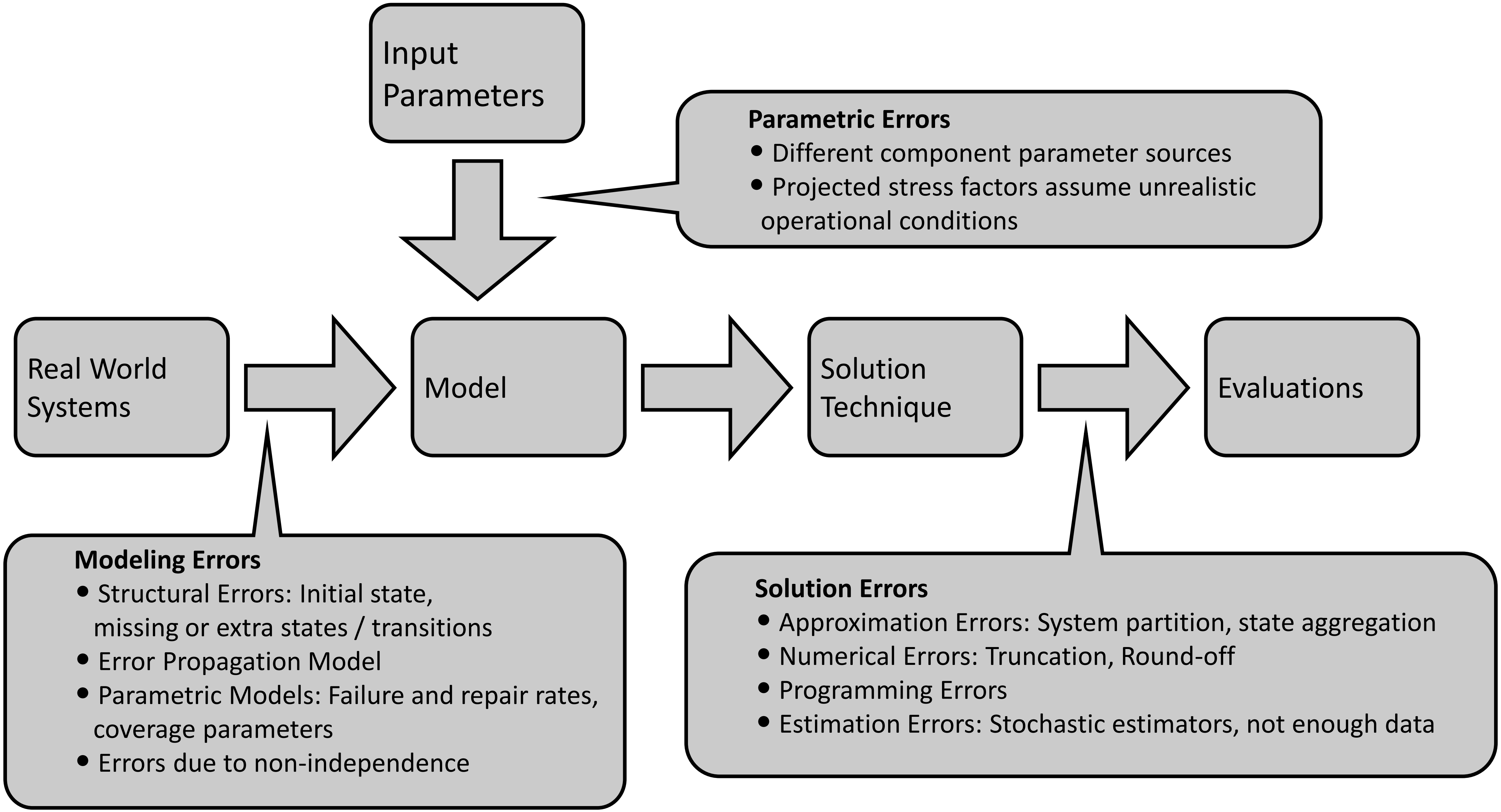
System structure
Fault classes
Failure rates
Fault handling procedures
Repair procedures
Success criteria

Model development

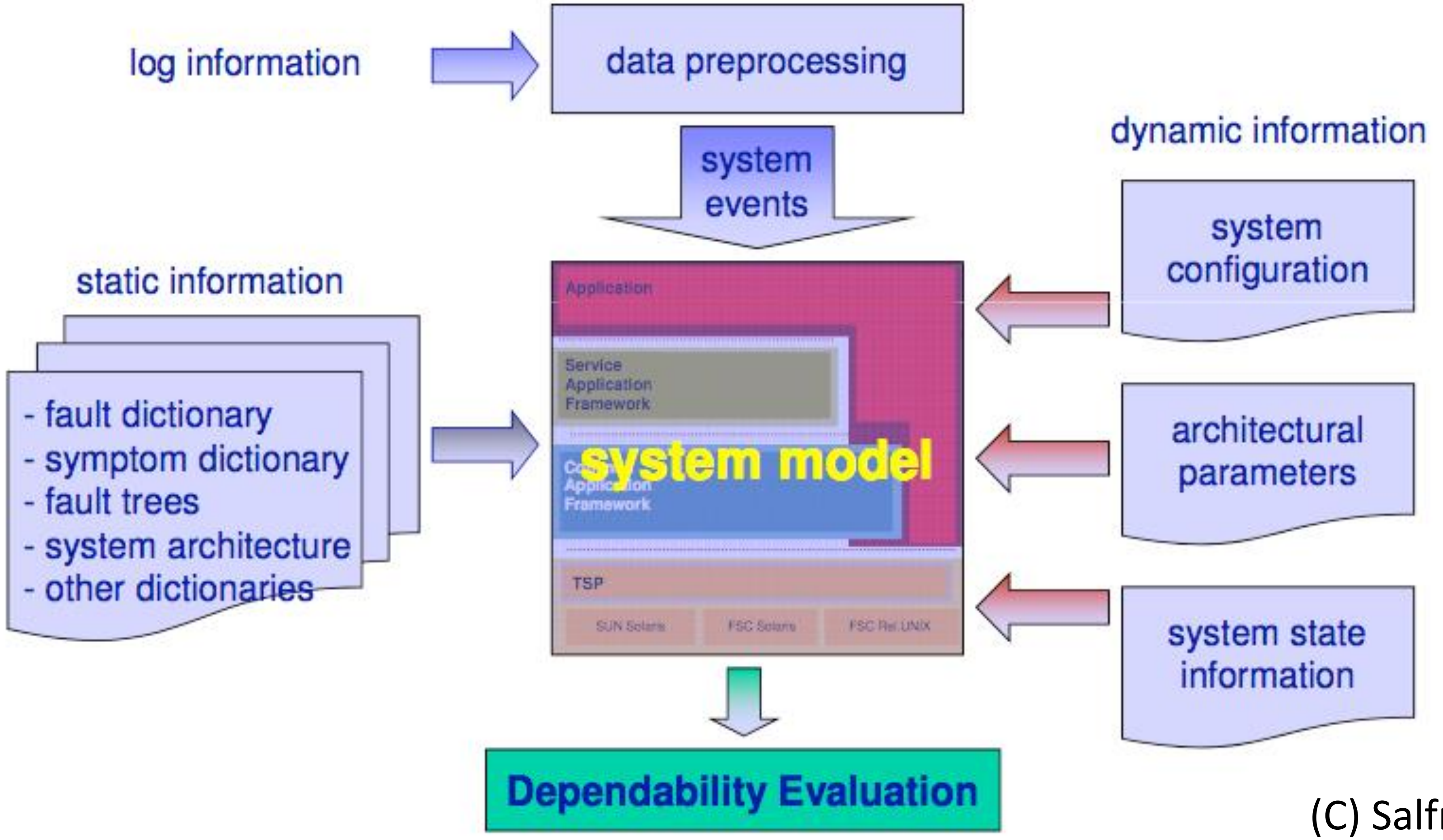
Model solution (combinatorial, simulation)

Dependability evaluation

Fault Coverage
Reliability
Availability
Life Cycle
MTTF / MTTR
Costs
…


Input
Parameters
Parametric Errors
• Different component parameter sources
• Projected stress factors assume unrealistic operational conditions
Real World Systems
Model
Solution Technique
Evaluations
Modeling Errors
• Structural Errors: Initial state, missing or extra states / transitions
• Error Propagation Model
• Parametric Models: Failure and repair rates, coverage parameters
• Errors due to non-independence
Solution Errors
• Approximation Errors: System partition, state aggregation
• Numerical Errors: Truncation, Round-off
• Programming Errors
• Estimation Errors: Stochastic estimators, not enough data

Runtime Dependability Evaluation

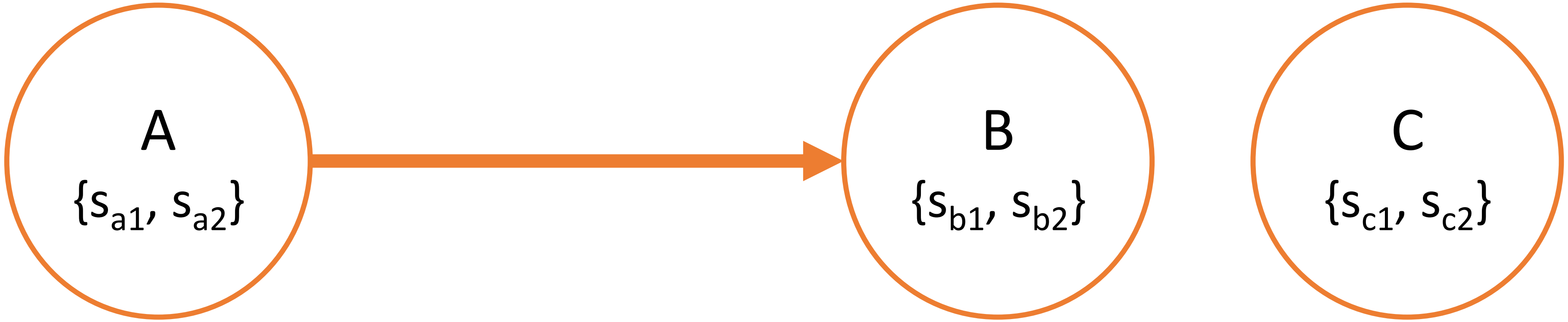


(C) Salfner

Bayesian Networks

- Encode **uncertain expert knowledge**
- Encode **causality relationships**
- Given a set of random variables, find **Joint Probability Distribution (JPD)**
- Bayesian approach:
  prior probability + likelihood → posterior probability
- Applications to dependability modeling
  - *Error propagation chains* as causality relationships:
    What is the probability of an overall error state, given prior per-component failure probabilities?
  - Online fault diagnosis
    What is the probability of observing the current system state, given that a processor is faulty/non-faulty?

Bayesian Networks



s_{a1}	s_{a2}
$P(A = s_{a1})$	$P(A = s_{a2})$

CPT (B)	s_{a1}	s_{a2}
s_{b1}	$P(B = s_{b1} \mid A = s_{a1})$	$P(B = s_{b1} \mid A = s_{a2})$
s_{b2}	$P(B = s_{b2} \mid A = s_{a1})$	$P(B = s_{b2} \mid A = s_{a2})$

s_{c1}	s_{c2}
$P(C = s_{c1})$	$P(A = s_{c2})$

$$\boldsymbol{JPD}: \quad P(x_1, \ldots, x_n) = \prod_{i}^{n} P(x_i \mid parents(X_i))$$

$$\text{e.g.}: \quad P(s_{a1} \wedge s_{b1} \wedge s_{c1}) = P(s_{a1}) P(s_{b1} | s_{a1}) P(s_{c1})$$

Example: Fault Tree → Bayesian Network